Technical Comments

Comment on "Wind-Tunnel Simulation of Store Jettison with the Aid of an Artificial Gravity Generated by Magnetic Fields"

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Nomenclature

A = $C_{L_\alpha}\gamma P/2M^2K_SL^2/W$
a = ratio of nonaerodynamic produced accelerations to gravitational acceleration
A' = $C_L\gamma P/2M^2K_SL^2/W$
B = $C_D\gamma P/2M^2K_SL^2/W$
C = $C_{M_\alpha}\gamma P/2M^2K_SL^2L/I$
C' = $C_\varphi\gamma P/2M^2K_SL^2L/I$
C_D = drag coefficient
C_L = lift coefficient
C_{L_α} = lift curve slope
C_φ = moment coefficient in roll
D_1 = $-Ag/2V + [(Ag/2V)^2 + Cg]^{1/2}$
D_2 = $-Ag/2V - [(Ag/2V)^2 + Cg]^{1/2}$
d = store mass density
e = base of natural logarithms
g = acceleration of gravity
I = store moment of inertia = $dK_VL^3K_rL^2$
K_r = radius of gyration coefficient of store = R^2/L^2
K_S = area coefficient = S/L^2
K_V = volume coefficient = V_S/L^3
L = characteristic length
M = freestream Mach number
P = freestream static pressure
R = radius of gyration of store in pitch
S = characteristic area of store
t = time
V = air flow speed
V_r = a velocity relative to store
V_S = characteristic volume of store
W = store weight = dK_VL^3
x = coordinate in the direction of air flow
x' = x/L
y = coordinate in the vertical direction
y' = y/L
α = angle of attack (Fig. 1)
γ = ratio of specific heats
γ_p = flightpath angle
θ = pitch angle (Fig. 1)
ρ = freestream density
φ = roll angle

Subscripts

0 = initial conditions

THE discussion of Ref. 1 concerning simulation of store jettison contains many simplifying assumptions. Some of these are implicit in the analysis and, in addition, some appear to be arbitrary. It is noted that the analysis reported in Ref. 2 appearing in the bibliography of Ref. 1, is similar in this respect. The purpose of this comment is to present an analysis which is sufficiently general to exhibit most of the interesting aspects of the mechanics of store separation and which will contain all assumptions explicitly. The effect of the assumptions on the simulation problem can then be readily determined.

Store separation is, in general, a most complex mechanics problem in that it can be considered at least a six-degree-of-freedom dynamics problem involving at least two bodies. If one or both bodies are controlled, additional degrees of freedom must be considered. Also, the terms store separation or jettison are only representative as the problem ranges from supply drops through crew member ejection. Most problems can be restricted to short time intervals after separation, but an extreme example of the opposite case has been the occurrence of fighter aircraft colliding with their own 20 mm shells during pullups from practice strafing passes. Thus, the trajectory of the parent aircraft cannot always be ignored.

It is assumed that a three-degree-of-freedom model will exhibit the general nature of the mechanics problem from the point of view of simulation. The equations for the motion of the store can be written

$$\ddot{y} = (A\alpha + a)g \tag{1}$$

$$\ddot{x} = Bg \tag{2}$$

$$\ddot{\theta} = C\alpha g \tag{3}$$

$$\alpha = \theta - \dot{y}/V \tag{4}$$

Several simplifying assumptions are required to obtain these equations. Referring to the nomenclature and Fig. 1, it is seen that the dynamic pressure is taken as $\frac{1}{2}\rho V^2$ rather than $\frac{1}{2}\rho V_r^2$ so that the influence of $\dot{x}$ and $\dot{y}$ is ignored. Equations (1) and (2) show that the flightpath angle γ_p is taken to be so small that $\cos\gamma_p = 1$ and $\sin\gamma_p = 0$. It is noted that Eq. (2) contains the assumption that the drag coefficient is independent of the angle of attack. It is undesirable to make this assumption, but it is necessary for analysis, as otherwise the equations become nonlinear. The use of Eqs. (1–4) to describe the problem further implies that the parent vehicle is assumed to follow a rectilinear, unaccelerated flightpath. Finally, it is assumed that the store is rigid, uncontrolled, and unpowered.

In most instances there is no desire to simulate occurrences in time. It is rather desired to simulate the flightpath and

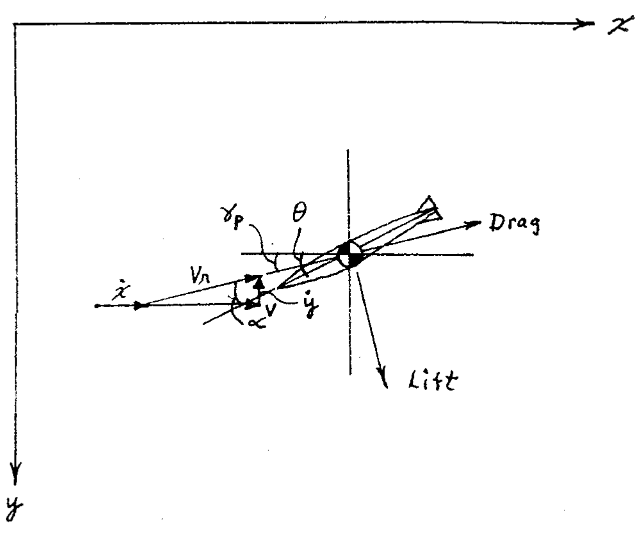

Fig. 1 Nomenclature.

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the rotations of the store. Thus, in this case, similarity requires the functions $y'(x')$ and $\theta(x')$ to be the same for the model and the flight vehicle. Here y' and x' are y and x nondimensionalized by a characteristic length. Differential equations for $y'(x')$ and $\theta(x')$ could be obtained by eliminating t from Eqs. (1–4). These equations and the boundary conditions would produce the similarity parameters. Alternatively, the equations can be solved by any of several methods and $t(x')$ substituted in $y'(t)$ and $\theta(t)$. This is done by first combining Eqs. (1–4) into

$$\ddot{\alpha} + Ag\dot{\alpha}/V - Cg\alpha = 0 \tag{5}$$

finding $\alpha(t)$, and then solving the $\ddot{y}'$ and $\ddot{\theta}$ equations to yield the general solutions

$$y'(t) = y'_0 + agt^2/2L + t[\dot{y}'_0 - Ag/L(\alpha_1/D_1 + \alpha_2/D_2)] + Ag/L[\alpha_1/D_1^2(e^{D_1t} - 1) + \alpha_2/D_2^2(e^{D_2t} - 1)] \tag{6}$$

$$\theta(t) = \theta_0 + t[\dot{\theta}_0 - Cg(\alpha_1/D_1 + \alpha_2/D_2)] + Cg[\alpha_1/D_1^2(e^{D_1t} - 1) + \alpha_2/D_2^2(e^{D_2t} - 1)] \tag{7}$$

$$t(x') = -\dot{x}'_0L/Bg + [(\dot{x}'_0L/Bg) + 2x'L/Bg]^{1/2} \tag{8}$$

assuming $x'_0 = 0$. If $t(x')$ is substituted into Eqs. (6) and (7), then $y'(x')$ and $\theta(x')$ are obtained. In this form, the boundary conditions at $t = 0$ are included. The presence of the exponential terms causes the similarity parameters to be the same as the parameters of the original differential equations and the boundary conditions. Thus, in general, no simplification is achieved. Some simplification is possible, however, if the exponentials are expanded in series. Retaining t in the equations, the first, second, and third approximations are then as follows:

1st approximation

$$y' = y'_0 + \dot{y}'_0t + agt^2/2L \tag{9}$$

$$\theta = \theta_0 + \dot{\theta}_0t \tag{10}$$

2nd approximation

$$y' = y'_0 + \dot{y}'_0t + \frac{agt^2}{2L} + \frac{Ag\alpha_0t^2}{2L} \qquad \alpha_0 = \theta_0 - \frac{\dot{y}'_0L}{V} \tag{11}$$

$$\theta = \theta_0 + \dot{\theta}_0t + Cg\alpha_0t^2/2 \tag{12}$$

3rd approximation

$$y' = y'_0 + \dot{y}'_0t + agt^2/2L + Ag\alpha_0t^2/2L + Ag\dot{\alpha}_0t^3/6L \tag{13}$$

$$\dot{\alpha}_0 = \dot{\theta}_0 - \ddot{y}'_0L/V$$

$$\theta = \theta_0 + \dot{\theta}_0t + Cg\alpha_0t^2/2 + Cg\dot{\alpha}_0t^3/6 \tag{14}$$

It is noted that in the first approximation the aerodynamic terms do not contribute to the motion except for the drag. The similarity parameters can be obtained by substituting for $t(x')$ from Eq. (8). Thus, for the first approximation

$$y' = y'_0 - \frac{\dot{x}'_0\dot{y}'_0L}{Bg} + \left[\left(\frac{\dot{x}'_0\dot{y}'_0L}{Bg}\right)^2 + \frac{2x'\dot{y}'_0{}^2L}{Bg}\right]^{1/2} + \frac{a}{B}\left\{\frac{\dot{x}'_0{}^2L}{Bg} + x' - \left[\left(\frac{\dot{x}_0{}^2L}{Bg}\right)^2 + \frac{2x'\dot{x}'_0{}^2L}{Bg}\right]^{1/2}\right\} \tag{15}$$

$$\theta = \theta_0 - \dot{\theta}_0\dot{x}'_0L/Bg + [(\dot{x}'_0\dot{\theta}_0L/Bg)^2 + 2x'\dot{\theta}_0{}^2L/Bg]^{1/2} \tag{16}$$

The similarity parameters are seen to be

y: $\quad y'_0 \quad a/B \quad \dot{x}'_0{}^2L/Bg \quad \dot{y}'_0/\dot{x}'_0$

θ: $\quad \theta_0 \quad \dot{\theta}_0/\dot{x}'_0 \quad \dot{x}'_0{}^2L/Bg$

Since y' and θ are coupled through the angle of attack, similarity for both θ and y' must be satisfied. If it is assumed that $\theta_0 = y'_0 = 0$, the similarity parameters for the problem are

$a/B \quad \dot{x}'_0{}^2L/Bg \quad \dot{y}'_0/\dot{x}'_0 \quad \dot{\theta}_0/\dot{x}'_0$

Table 1 Similarity parameters

Approximation	Parameters[a] (for $\theta = x'_0 = y'_0 = 0$)
1st	a/B $\quad \dot{x}'_0{}^2L/Bg$ $\quad \dot{y}'_0{}^2L/Bg$ $\quad \dot{\theta}_0{}^2L/Bg$
2nd	$(a + \alpha_0A)/B$ $\quad C\alpha_0L/B$ $\quad \dot{x}'_0{}^2L/Bg$ $\quad \dot{y}'_0{}^2L/Bg$ $\quad \dot{\theta}_0{}^2L/Bg$
3rd	a/B $\quad A/B$ $\quad C/B$ $\quad \dot{y}'_0L/V$ $\quad \dot{x}'_0{}^2L/Bg$ $\quad \dot{y}'_0{}^2L/Bg$ (or V^2/LgB) $\quad \dot{\theta}_0{}^2L/Bg$ $\quad (\dot{\alpha}_0{}^2L/Bg)$[a]

[a] This parameter contains all the other initial value parameters.

If $\dot{x}'_0 = 0$ the parameters are

$a/B \quad \dot{y}'_0{}^2L/Bg \quad L\dot{\theta}_0{}^2/Bg$

The parameters for the second and third approximations can be obtained in a similar fashion. These are given in Table 1. It is seen from Table 1 that in the third approximation there is already only one less similarity parameter than there are parameters in the Eqs. (1–4) plus the initial conditions. It is noted that only in the third and higher approximations is the angle of attack the same function of x' for both the model and the flight vehicle.

The similarity parameters can be examined in terms of the variables and constants associated with the air flow and the vehicle. Thus, except for the initial conditions, the first approximation requires only that a/B is constant. This can be written as

$$a/B = a/[C_D(\gamma PM^2/2)K_SL^2]/(dK_VL^3) = (\gamma M^2/2)(C_DK_S/K_V)(P/dLa) \tag{17}$$

The second approximation provides

$$\frac{\left[a + \left(\theta_0 - \frac{\dot{y}'_0L}{V}\right)\right]\left[\frac{C_{L\alpha}(\gamma P/2)M^2K_SL^2}{dK_VL^3}\right]}{(\gamma M^2/2)(C_DK_S/K_V)(P/dL)} = \left(\frac{\gamma M^2}{2}\frac{C_DK_S}{K_V}\frac{P}{dLa}\right)^{-1} + \left(\theta - \frac{\dot{y}'_0L}{V}\right)\left(\frac{C_{L\alpha}}{C_D}\right) \tag{18}$$

and

$$\frac{C_{M\alpha}(\gamma P/2)M^2K_SL^4[\theta_0 - (\dot{y}'_0L/V)]}{(dK_VL^3)(K_rL^2)(\gamma M^2/2)(K_SC_D/K_V)(P/dL)} = \left(\frac{C_{M\alpha}}{C_DK_r}\right)\left(\theta_0 - \frac{\dot{y}'_0L}{V}\right) \tag{19}$$

in addition to the initial conditions. Thus, in the first approximation, similitude would exist if M, C_DK_S/K_V and P/dLa were held constant between the model and the flight vehicle and the initial conditions were satisfied. In the second approximation, $(C_{L\alpha}/C_D)[\theta_0 - (\dot{y}'_0L/V)]$ and $(C_{M\alpha}/C_DK_r)[\theta_0 - (\dot{y}'_0L/V)]$ must be held constant, as well, for similitude. Thus, if the aerodynamic and dynamic coefficients are the same for both model and flight vehicle, then the initial angles of attack must be equal for similitude in the second approximation.

If all the aerodynamic forces are taken to be independent of the angle of attack, then the y and θ equations are uncoupled. It is more appropriate perhaps to examine the motion with θ replaced by φ, the roll angle, in this case. The equations are

$$\ddot{y}' = (A' + a)g/L \tag{20}$$

$$\ddot{x}' = Bg/L \tag{21}$$

$$\ddot{\varphi} = C'g \tag{22}$$

The general solutions in terms of x' are

$$y' = \frac{(A'+a)}{B}\left\{\frac{\dot{x}_0'^2 L}{Bg} - x' - \left[\left(\frac{\dot{x}_0'^2 L}{Bg}\right)^2 + \left(\frac{\dot{x}_0'^2 L}{Bg}\right)2x'\right]^{1/2}\right\} + \left\{\frac{-\dot{x}'_0\dot{y}'_0 L}{Bg} + \left[\left(\frac{\dot{x}'_0\dot{y}'_0 L}{Bg}\right)^2 + 2x'\,\frac{\dot{y}_0'^2 L}{Bg}\right]^{1/2}\right\} + y'_0 \quad (23)$$

$$\varphi = \frac{C'L}{B}\left\{\frac{\dot{x}_0'^2 L}{Bg} - x' - \left[\left(\frac{\dot{x}_0'^2 L}{Bg}\right)^2 + \left(\frac{\dot{x}_0'^2 L}{Bg}\right)2x'\right]^{1/2}\right\} + \left\{-\frac{\dot{\varphi}_0\dot{x}'_0 L}{Bg} + \left[\left(\frac{\dot{\varphi}_0\dot{x}'_0 L}{Bg}\right)^2 + 2x'\left(\frac{\dot{\varphi}_0^2 L}{Bg}\right)\right]^{1/2}\right\} + \varphi_0 \quad (24)$$

The similarity parameters are seen to be

$$y':\quad \frac{(A'+a)}{B} \qquad \frac{\dot{x}_0'^2 L}{Bg} \qquad \frac{\dot{y}_0'^2 L}{Bg} \qquad y'_0$$

$$\varphi:\quad \frac{C'L}{B} \qquad \frac{\dot{x}_0'^2 L}{Bg} \qquad \frac{\dot{\varphi}_0^2 L}{Bg} \qquad \varphi_0$$

In terms of air flow and vehicle variables, the parameters for similarity of both y' and φ are

$$\frac{(A'+a)}{B} = \frac{[C_L(\gamma P/2)M^2K_SL^2/dK_VL^3 + a]}{[C_D(\gamma P/2)M^2K_SL^2/dK_VL^3]} = \frac{C_L}{C_D} + \frac{1}{(\gamma M^2/2)(C_DK_S/K_V)(P/dLa)} \quad (25)$$

$$\dot{x}_0'^2L/Bg = \dot{x}_0'^2/(\gamma M^2/2)(C_DK_S/K_V)(Pg/dL^2) \quad (26)$$

$$\dot{y}_0'^2L/Bg = \dot{y}_0'^2/(\gamma M^2/2)(C_DK_S/K_V)(Pg/dL^2) \quad (27)$$

$$\dot{\varphi}_0^2L/Bg = \dot{\varphi}_0^2/(\gamma M^2/2)(C_DK_S/K_V)(Pg/dL^2) \quad (28)$$

$$\frac{C'L}{B} = \frac{0.5C\varphi\gamma PM^2K_SLL^2L}{(dK_VL^3K_rL^2)(0.5C_D\gamma PM^2K_SL^2/dK_VL^3)} = \frac{C\varphi}{C_D} \quad (29)$$

It is seen that these are the same similarity parameters obtained in the "light model method" analysis of Ref. 2 for $a = 1$. In this reference the analysis contains the assumption that $\gamma M^2/2$, C_L/C_D, C_DK_S/K_V and $C\varphi/C_D$ are constant for model and flight vehicle. In addition, the assumption of P/d = const is made. It is evident from Eq. (25) that thus an exact simulation cannot be obtained since from the foregoing, P/dL = const must also be true, i.e., the scale L would be one to one. In fact, considering Eq. (25) and Eqs. (26–28), it is evident that for exact similitude L = const is required if any of $\dot{x}'_0$, $\dot{y}'_0$ or $\dot{\varphi}'_0$ are nonzero. The major portion of the discussion of Ref. 2 is directed toward approximate similitude studies, in recognition of this circumstance.

If a is a variable then it is evident that exact similitude can be achieved in this case. It is seen from Table 1 that the first, second, and third approximations produce similarity parameters that can be matched exactly for the model and flight vehicle if a is a variable. However, the flow or flight velocity V appears in the third approximation and, as noted in Ref. 1, if the Mach number is held constant, V varies as the square root of the temperature. This variable has a relatively limited range in many experimental facilities.

References

[1] Covert, E. E., "Wind-tunnel simulation of store jettison with the aid of an artificial gravity generated by magnetic fields," J. Aircraft **4**, 48–51 (1967).

[2] Sandahl, C. A. and Faget, M. A., "Similitude relations for free-model wind tunnel studies of store-dropping problems," NACA TN 3907 (January 1957).